



Calculation of the Two-Neutron Separation Energies of Even-Even $^{36-58}\text{Ca}$, $^{50-78}\text{Ni}$, $^{102-138}\text{Sn}$ and $^{182-220}\text{Pb}$ Nuclei by an Artificial Neural Network Model

Bir Yapay Sinir Ağı Modeli Yardımıyla $^{36-58}\text{Ca}$, $^{50-78}\text{Ni}$, $^{102-138}\text{Sn}$ ve $^{182-220}\text{Pb}$ Çekirdeklerinin İki-Nükleon Ayırma Enerjilerinin Hesaplanması

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Abstract

In this study, an Artificial Neural Network (ANNs) model was developed in order to calculate the two-neutron separation energies (S_{2n}) for the even-even $^{36-58}\text{Ca}$, $^{50-78}\text{Ni}$, $^{100-138}\text{Sn}$ and $^{182-220}\text{Pb}$ nuclei with the magic proton numbers, 20, 28, 50 and 82, respectively. The obtained results were compared with the Liquid Drop Model (LDM), Relativistic Mean Field Theory (RMFT) and the experimental results.

Keywords: Liquid drop model, Neural network, Relativistic mean field theory

Öz

Bu çalışmada, sırasıyla, 20, 28, 50 ve 82 sihirli sayıda protona sahip $^{36-58}\text{Ca}$, $^{50-78}\text{Ni}$, $^{100-138}\text{Sn}$ ve $^{182-220}\text{Pb}$ çift-çift çekirdeklerinin iki-nötron ayırma enerjilerini hesaplanmak için bir Yapay Sinir Ağları (YSA) modeli geliştirildi. Elde edilen sonuçlar, Sıvı Damlası Modeli (SDM) ve Rölativistik Ortalama Alan Teori (ROAT) sonuçları ile karşılaştırıldı.

Anahtar Kelimeler: Sıvı damlası modeli, Yapay sinir ağı, Rölativistik ortalama alan teorisi

1. Introduction

Microscopic models have been extensively used to describe the global nuclear properties such as nuclear masses, binding energies and nuclear separation energies. These models include the effective interactions in the mean field Hamiltonian of nucleons. Most popular microscopic models developed for even-even nuclei are the Relativistic Mean Field (RMF) and Hartree-Fock Bogoliubov (HFB) theories. The RMF model has been very successful in the general description of ground states properties of even-even nuclei (Ring 1996). Effective interactions such as Skyrme-like forces are successfully used in the Hartree-Fock Bogoliubov theory to characterize the global properties of even-even nuclei (Bennaceur and Dobaczewski 2005).

Nuclear binding energy as a fundamental ground state property of a nucleus, which can be defined as the energy

holding nucleons together in a nucleus or the amount of energy required to separate a nucleus into its components that are nucleons. Binding energy is also an observable quantity and reflects the interactions among the nucleons inside the nucleus. Two-proton and two-neutron separation energies are the other observables, which are subtracted from binding energies, present considerable information about the nuclear structure. Therefore, the exactly prediction of binding energy is important. None of the present microscopic theories is a unique satisfactory theory concerning the attractive interaction, which is responsible for nuclear binding energy. For this reason, a simple and reliable model for nuclear binding energy can provide information for neutron rich nuclei. In the previous studies, nuclear binding energies, two-proton, two-neutron separation energies of neutron-rich light and medium even-even nuclei, were investigated theoretically in the Hartree-Fock-Bogoliubov method using different Skyrme force parameters (Aytekin 2012, Aytekin and Artun 2013).

As explained in above studies for one or two-nucleon separation energies play important role in identifying nuclear

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structure of nuclei, which have proton and neutron excess (drip-line). The nucleon separation energies are mostly related to the atomic masses or binding energies of nuclei. Artificial neural networks (Athanasopoulos et al. 2004) are also used to describe the nucleon separation energies as well the finite range droplet model and the Hartree-Fock-Bogoliubov (HFB) model. Recently, ANNs have been used in many fields of nuclear physics given by the ref. (Bayram et al. 2014) and the other references therein.

The systematic study for proton and neutron separation energies is essential to investigate the nuclear structures of neutron and proton drip-line nuclei. The energy spent in removing of two fermions from strongly correlated identical fermion system must be a good indicator of the stability of the system (Anghel et al. 2009). Moreover, the separation energies of two fermions have much higher values for nucleonic systems with even-even nuclei than the odd number ones. These are the general characters, which can be seen along the isotopic chains (Wang et al. 2012). The aim of this study is to test a developed ANNs model for investigation of two-neutron separation energies $S_{2n}(A, Z)$ of the even-even nuclei $^{36-58}\text{Ca}$, $^{50-78}\text{Ni}$, $^{102-138}\text{Sn}$ and $^{182-220}\text{Pb}$, which have the magic proton numbers, 20, 28, 50 and 82, respectively. The obtained results have been compared with the results of LDM and RMF models and experimental.

2. Theoretical Framework

ANNs are inspired from biological neural systems, which are used for modelling of complex processes or the processes that are not completely understood (Hornik et al. 1989). ANNs are formed by the neurons which are basic processing units of the network as shown in Figure 1. These neurons are connected to each other by the “synapses”. These synapses take the values from their input (x_i) and multiply them by specific constants called weights (w_i), which are determined by a process known as “training”. Firstly, a neuron sums all the signals that come from its synapses, and then passes the sum through an activation function (φ_i) which has threshold values q_i . The output of the activation function is sent to the next layer (output function) as shown in Figure 1 (Diamantaras and Kung 1996). The activation function $\varphi_i = f(x_i, w_i)$ represents main calculation performed by a biological neuron and the output function $y_i = f(\varphi_i)$ corresponds to the overall activity transmitted to the next neuron (Debes et al. 2005).

The activation function $\varphi_i = f(x, w_i)$ and the output function $y_i = f(\varphi_i)$ are summed up with the term transfer functions.” (Debes et al. 2005).

A graphical interface allows one to create an ANNs. Therefore in this study, we used a graphical interface by a MATLAB code to create an ANNs. We made the code

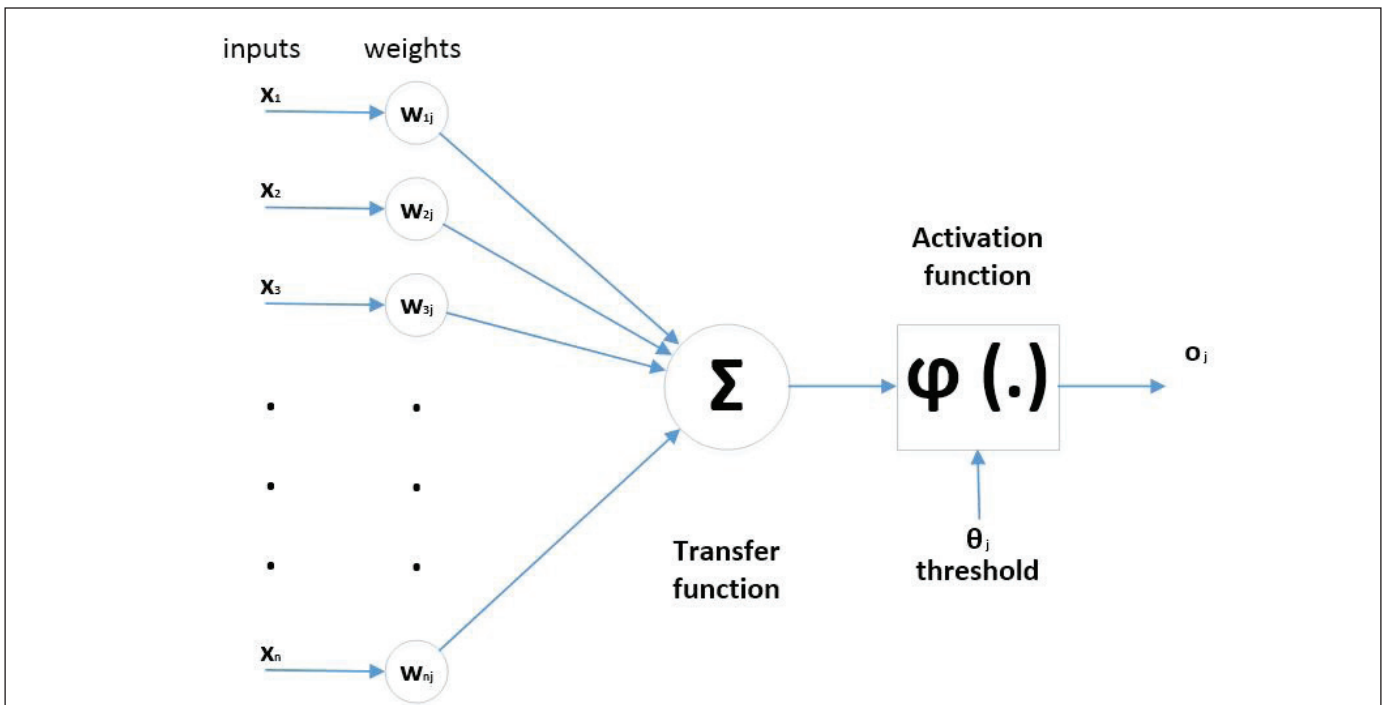


Figure 1. ANNs are formed by neurons, which are the basic units of network (Diamantaras and Kung 1996).

to be trained for nuclear binding energies by inserting the neutron (N) proton (Z) numbers in the code for randomly selected nuclei. During the course of learning, 200 different even-even nuclei were used. After the training of the code, the obtained data was fitted to the experimental data by the Neural Net Fitting model. In this fitting process, the Bayesian network method was chosen. "A Bayesian network is a representation of a joint probability distribution of a set of random variables with a possible mutual causal relationship" (Horný 2014). The Bayesian network method was selected for the best fitting. After this selection, the process was tested until the best agreement with the experimental results to be evaluated.

3. Results and Discussion

The S_{2n} energies for the even-even nuclei $^{36-58}\text{Ca}$, $^{50-78}\text{Ni}$, $^{100-138}\text{Sn}$ and $^{182-220}\text{Pb}$ were calculated in the ANNs model by subtracting the adjacent binding energies as

$$S_{2n}(Z, N) = BE(Z, N) - BE(Z, N - 2).$$

The results for $S_{2n}(Z, N)$ were compared with the results of LDM, RMFT and the experimental results in Figure 2.

As it is seen from the Figure 2, the ANNs results of $S_{2n}(Z, N)$ for $^{36-58}\text{Ca}$, $^{50-78}\text{Ni}$, $^{102-138}\text{Sn}$ and $^{182-220}\text{Pb}$ nuclei generally more close to the experimental results than that of the other models RMFT and LDM. Figure 2A shows the comparison of the calculated $S_{2n}(Z, N)$ results obtained in the ANN

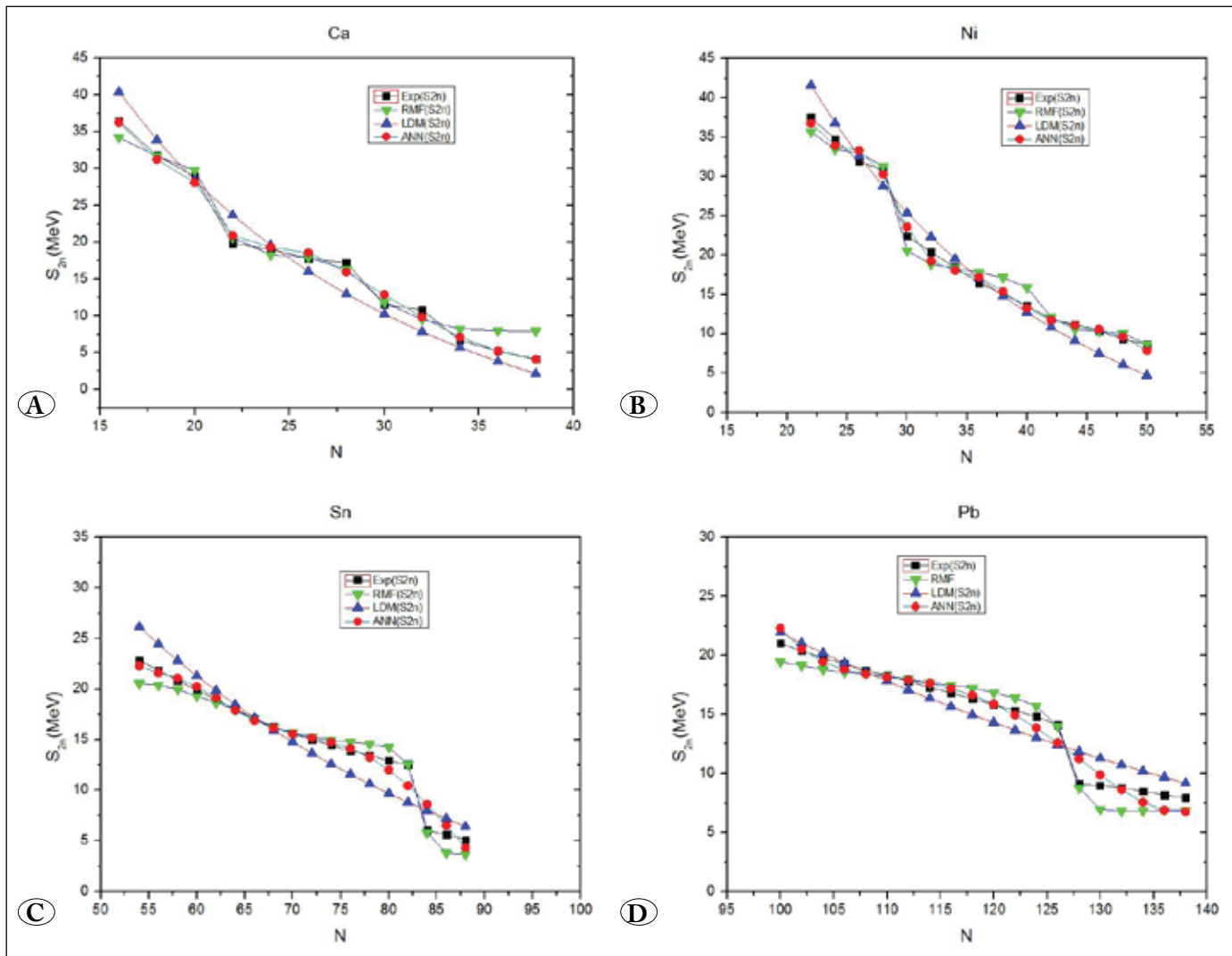


Figure 2. Comparison of the two-neutron separation energies for (A) $^{36-58}\text{Ca}$, (B) $^{50-78}\text{Ni}$, (C) $^{100-138}\text{Sn}$ and (D) $^{182-220}\text{Pb}$ nuclei obtained in the ANNs model with the experimental (Wang et al. 2012), LDM (http://bcs.whfreeman.com/webpub/Ektron/Tipler%20Modern%20Physics%206e/More%20Sections/More_Chapter_11_1-Liquid-Drop_Model_and_the_Semiempirical_Mass_Formula.pdf) and RMFT (Lalazissis et al. 1999). The graphs represent the variations of the S_{2n} values with the neutron numbers (N).

model with that of the RMFT, LDM and experimental for the nuclei $^{36-58}\text{Ca}$. The agreements of the ANNs results for the isotopes $^{36,44,52-58}\text{Ca}$ with the experimental results ($N=16, 24, 32-38$) more close to the experimental results than that of the RMFT and LDM. Deviations of these results from the experimental ones are in the range of 0.17 MeV-1.35 MeV. On the other hand, the results for the isotopes $^{38,42, 46-50}\text{Ca}$ ($N=18,22, 26-30$) in the RMFT and the result of ^{40}Ca ($N=20$) in the LDM are better than that of the ANNs model.

We compared the results of $^{50-78}\text{Ni}$ obtained in the ANNs model with the results of RMFT and LDM in Figure 3B. The results for the isotopes $^{50,56-60,68-72,76}\text{Ni}$ ($N=22, 28-32, 40-44, 48$) in the ANNs model more close to the experimental results than that of the RMFT and LDM. These results are deviates from the experimental ones in the range of 0.080 MeV-1.12 MeV. On the other hand, the results of RMFT for the isotopes $^{54,62,74,78}\text{Ni}$ ($N=26, 34, 46, 50$) and the results of LDM for the isotopes $^{52,54,64,66}\text{Ni}$ ($N=24, 26, 36, 38$) are better than the ANNs results.

We compared the S_{2n} results of $^{102-138}\text{Sn}$ ($N=52, 88$) isotopes obtained in the ANNs model with the results of RMFT and LDM models in Figure 2C. The results for the isotopes $^{102-114}\text{Sn}$ ($N=52-64$) and $^{120-130}\text{Sn}$ ($N=70-80$) and $^{136, 138}\text{Sn}$ ($N=86, 88$) more close to the experimental results than that of the RMFT and LDM. Deviations of the ANNs results for these nuclei from the experimental results are in the range of 0.010 MeV-1 MeV. On the other hand, only the results of RMFT for the isotopes $^{118,132,134}\text{Sn}$ ($N=68, 82, 84$) isotopes are better than the ANNs results. In the case of LDM, only the result for ^{116}Sn ($N=66$) is better than the results of both the ANNs and RMFT models.

We compared the S_{2n} results of the isotopes $^{182-220}\text{Pb}$ ($N=100, 138$) obtained in the ANNs model with the results of RMFT and LDM as seen in Figure 2D. The results of the isotopes $^{184, 186, 194-202, 212-218}\text{Pb}$ ($N=102, 104, 112-122, 130-136$) in the ANNs model generally more close to the experimental results than that of the RMFT and the LDM. Deviations of these results from the experimental ones are in the range of 0.050 MeV-1.3 MeV. On the other hand, the results for the isotopes $^{192, 208, 210, 220}\text{Pb}$ ($N=110, 124-128, 138$) the RMFT and for $^{182,188,190}\text{Pb}$ ($N=100, 106, 108$) in the LDM more close to the experimental ones than that of the ANNs model.

4. Conclusion

In this study, we calculated the binding energies of even-even nuclei $^{36-58}\text{Ca}$, $^{50-78}\text{Ni}$, $^{102-138}\text{Sn}$ and $^{182-220}\text{Pb}$ by a developed

ANNs model. Using these binding energies we calculated the two-neutron separation energies for the same nuclei and then compared them with that of the experimental, RMFT and LDM results. Our analyses have shown that the results obtained by the ANNs model are generally agreement with the experimental results in the large isotope ranges. Also, we conclude that the results obtained in the ANNs model averagely more close to the experimental results than that of the RMFT and LDM.

5. References

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